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Referee's report on the PhD dissertation by

mgr Adam Glos

Supervisor: dr hab. Jarosław Adam Miszczak

"Application of graph theory in quantum computer science"

Institute of Theoretical and Applied Informatics, Polish Academy of Sciences

The topic of the thesis by Adam Glos is the dynamics of quantum walks on graphs and their possible application for search problems via graph theory methods. The problem of quantum walks has attracted much interest in recent years as they are effective tools in quantum information theory. The main aim of the present work is to generalize existing quantum walk models to directed graphs and to extend the results on spatial search by continuous-time quantum walks (CTQWs) on Erdős-Rényi graphs as well as other, heterogeneous graphs.

The dissertation contains a concise introduction (Chapter 2) to CTQWs, outlining the mathematical formulation and also providing an overview of the relevant results from the literature. The following 4 chapters contain the main results of the dissertation. In Chapter 3 the author presents the central results concerning a novel definition of a CTQW on a directed graph maintaing fast propagation. This superdiffusive quantum stochastic walk is an interesting and nontrivial construction, since the problems of spontaneous moralization and premature localization had to be overcome. In Chapter 4 the new construction (NGQSW) is analysed in detail in different situations and compared to other models, especially important theorems are derived for their convergence. In Chapter 5 the author returns to the problem of original CTQWs on undirected graphs and their application for search problems. First, some recent results by Chakraborty et al. are considerably improved, refined and extended. The Laplacian matrix is shown to be superior to the adjacency matrix for search problems in this case. In Chapter 6 the efficiency of CTQW based spatial search on heterogeneous and complex graphs is examined. Especially, the Chung-Lu and the Barabási-Albert graphs models are considered. The latter model is very important, since it is a paradigmatic example for complex networks. The presented analytical and numerical results convincingly show that a quadratic speedup can be achieved also here, eventhough one must carefully take into account some specific details, such as the degree of the node one searches for. The problem of optimal measurement time is also nontrivial, but may be circumvented in the considered cases. This question is only partially answered in the dissertation.

The results of the present dissertation consist of both analytically proven theorems and extensive numerical simulations. The Julia package developed for the quantum stochastic walk analysis has also been published in Computer Physics Communications. The thesis is clearly written in a good scientific style, I did not find errors. A large part of the presented results have been published in high quality scientific journals (2 Quant. Inf. Comp., Quant. Inf. Proc., Phys. Rev. A, J. Phys. A: Math. Theor.). Moreover, there are several other high quality publications co-authored by the applicant, which contain results not included in the present thesis. In my opinion, the general scientific quality of this PhD work is internationally competitive and represents excellent research.

I have a few remarks/questions about possible generalizations (out of curiosity, these do not concern my positive opinion about the thesis):

1., In the first part of the thesis a very elegant generalization of the definition of a CTQW is presented for directed graphs. It was not clear to me, whether there are some interesting applications where one could use these walks?

2., The applicant considers various graphs, I found especially interesting the case of complex, Barabási-Albert type graphs. Would it be possible to apply or generalize some of the presented ideas to graphs representing fractal structures, like the Sierpiński triangle?

3., Is there some fundamental reason behind the difference between the Laplacian vs adjacency matrix behavior? Are these the only interesting possibilities to construct the dynamics or one could search for other matrices as well?

In summary, the author achieved interesting and important results and considerably improved our understanding of different continuous-time quantum walk models on various graphs. Therefore, I recommend the dissertation for defense without hesitation and would suggest the best mark for it.

Tamás Kiss